Using Chaos Theory to Identify the Dynamic States of an Urban Road Network for Traffic Control

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This presentation gives the results of the first stage of my research for a PhD at Newcastle University and therefore it is currently not published. Please treat the content sensitively and contact Abraham Narh (a.t.narh@ncl.ac.uk) for any detailed information.
Outline of Presentation

- Background
- Challenges of UTC
- Chaos Theory
- Lyapunov Exponents
- Examples of Lyapunov Profiles
- Conclusion

The Butterfly-Effect

Phase Portrait
Background

- Increase in road traffic and travel demand
  >> congestion, delays, accidents
  >>> detrimental impact on environment

- Road transport >> 90% of CO₂ emissions from the transport sector (DfT, 2004)

- 89% of delays due to congestion in urban areas.

- By 2025 if left unchecked, will cost an extra £22 billion worth of time in England alone (Eddington, 2006)

- By 2025, 13% of traffic in congestion will be subject to start-stop conditions (Eddington, 2006)
Challenges of UTC

- **UTC Systems e.g. SCOOT**
  - perform well in under-saturated traffic conditions,
  - manage queues on a local level
  - Assume ‘fixed plans’ during over-saturated conditions

- Unable to reliably forecast the onset of congestion
  - in real-time
  - on a local or strategic level

- Chaos Theory has potential to help tackle this challenge

- Allowing effective preventative action by adjusting in advance traffic signal settings appropriately
Chaos Theory
Finding the Lag Time ($\tau$)

- The autocorrelation coefficient at lag $\tau$ is given by:

$$C(\tau) = \frac{\sum_{i=1}^{N-\tau}[x(i) - \bar{x}][x(i + \tau) - \bar{x}]}{\sum_{i=1}^{N}[x(i) - \bar{x}]^2}$$

where:
- $\bar{x}$ is the mean of observed data series;
- $x(i)$ is the preceding time observation;
- $x(i + \tau)$ is the observation at the lagged time ($\tau$)

- Plot the autocorrelation coefficient against the lag $\tau$ to establish the solution that is independent $C(\tau) = 0.4$

- For occupancy lag $\tau$ was 25-33 minutes for different months of the year
Finding the Dimension

- **Correlation Dimension/Embedding Dimension:**

Consider two points in the reconstructed phase space:

\[ X(j) = x(j), x(j + \tau), x(j + 2\tau), \ldots \ldots \ldots \ldots \ldots \ldots x(j + (m - 1)\tau) \]  
\[ \text{…………………………} (1) \]

\[ X(i) = x(i), x(i + \tau), x(i + 2\tau), \ldots \ldots \ldots \ldots \ldots \ldots x(i + (m - 1)\tau) \]  
\[ \text{…………………………} (2) \]

Let \( r_{ij} (m) \) denote the distance between them, so that:

\[ r_{ij}(m) = \| X_i - X_j \| \]

- **For occupancy the dimension was 3**
Lyapunov Exponent

The instantaneous Lyapunov Exponent is given by:

\[
\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \left| \frac{dx(X_0, t)}{dX_0} \right|
\]

where:
- \( dX_0 \) is the initial separation between two points;
- \( dx(X_0, t) \) is the separation after a time lapse \( t \)

Lyapunov exponent is a measure of congestion performance
Identifying states of congestion

- Requires time series data (e.g. from Motes, Bluetooth, ANPR, SCOOT) to estimate the Lyapunov Exponent

- Detect chaotic behaviour using the Lyapunov Exponent ($\lambda$):
  - If $\lambda < 0$, traffic is asymptotically stable (No congestion);
  - If $\lambda = 0$, steady traffic state i.e. exhibits Lyapunov stability;
  - If $\lambda > 0$, chaotic traffic state (emergence of congestion)
Results: Lyapunov Profiles

- Unstable State
- Asymptotic Stable State
- Steady Stable State
Results: Lyapunov Profiles

- Lyapunov Profile (from 0600 to 1159, 11-June-2002)

Unstable State

Asymptotic Stable State
Results: Lyapunov Profiles

- Lyapunov Profile (from 1200 to 1859, 11-June-2002)
Next Step

- Developed Lyapunov Exponent as an indicator of the on-set of congestion for a link over time
- Next step is to develop a method to establish relationship between the cause and effect spatially over time across the network
- Data sources that give an area-wide view of the evolution of congestion will enable traffic to be managed to avoid SCOOT junctions becoming over-saturated
Conclusion

- For SCOOT link occupancy at 20 second sampling interval there are slices (lags of 25-33 minutes duration) of time that are independent.
- Blocks of 3 slices behave independently.
- Lyapunov Exponent can enable traffic managers to forecast the onset of congestion in real-time over the network.
- Chaos Theory to enable traffic to be managed to avoid SCOOT junctions becoming over-saturated.
Thank you for listening

Any Questions?

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